LOCALIZATION ERRORS DUE TO THE IONOSPHERE, THE ALTITUDE OF THE BALLOONS, AND THE INACCURACY OF THE EPHEMERIDES

CENTRE NATIONAL D'ETUDES SPATIALES

Translation of Memorandum No. 8/65, Centre National d'Etudes Spatiales, 368, March 1966, 39 pages.
Toulouse

(NASA-TT-F-14701) LOCALIZATION ERRORS DUE TO THE IONOSPHERE, THE ALTITUDE OF THE BALLOONS, AND THE INACCURACY OF THE EPHEMERIDES (NASA) Sep. 1972 42 p CSCL 17G G3/21
N73-11678
Unclas
46401

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 SEPTEMBER 1972

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
U S Department of Commerce
Springfield VA 22151

/ 3

ADVANCED SCHOOL OF AERONAUTICS

AUTOMATION RESEARCH CENTER

CNES 368 - MARCH 1966

MEMORANDUM NO. 8/65

LOCALIZATION ERRORS DUE TO THE IONOSPHERE, THE ALTITUDE OF THE BALLOONS, AND THE INACCURACY OF THE EPHEMERIDES

I

TABLE OF CONTENTS

		Page Number
	INTRODUCTION	2
I.	ERRORS DUE TO IONOSPHERE	3
	1.1 Brief Review	3
	1.2 Formulation	8
	1.2.1 Absolute errors of distance due to ionospheric effect	8
	1.2.2 Calculation of elevation	9
	1.2.3 Ionospheric errors	10
	1.3 Results	11
	1.3.1 Influence of base position	11
	1.3.2 Influence of time between interrogation	11
	1.3.3 Influence of distance from the trace	12
	1.3.4 Influence of total electron content	13
	1.4 Influence respectively of the Ionosphere and of the Electronic Errors	13
	1.5 Conclusion	13
II.	ERRORS DUE TO VARIATIONS OF ALTITUDE OF THE BALLOONS	14
	2.1 Brief Review	14
	2.2 Formulation	15
	2.3 Results	19
	2.3.1 Influence of base position	19
	2.3.2 Influence of distance from the trace	20
	2.3.3 Influence of interval between interrogation	20
	2.4 Conclusion	21
III.	ERRORS DUE TO INACCURACIES IN THE EPHEMERIDES	21
	3.1 Brief Review	21
	3.2 Formulation	23

TABLE OF CONTENTS
(Continued)

	Page Number
3.3 Results	26
3.3.1 Influence of base position	26
3.3.2 Influence of distance from the trace	26
3.3.3 Influence of the interval between interrogation	26
3.4 Conclusion	27

/

INTRODUCTION

This memorandum is a continuation of memoranda No. 6 and 7 respectively covering errors of localization due to electronic errors of measurement and such errors due to the wind.

The memorandum examines the influence of other sources of error of distance which may affect localization. Only "isolated" influences are given here in principle and a future memorandum will synthesize the findings of this memorandum and of the earlier memoranda No. 6 and 7.

- - - - - - - - - - -

I. ERRORS DUE TO IONOSPHERE

1.1 Brief Review

This brief review is based on the memorandum of J.C. Thoin "Project Eole, an evaluation of the accuracy in the localization of the balloons and the role of the ionosphere as criterion for selecting two methods of localization" on which this paragraph leans heavily. We are introducing this study here for reasons of greater clarity and homogeneity.

The errors of localization considered in memorandum No. 6 were basically due to electronic errors of measurement. In addition to these errors which were evaluated by CFTH ("Results of Measurement on the "Eole" Model in October 1965), it is necessary to take into account the effect due to the ionosphere whose influence is the subject of this section. Measurement of the distance satellite-balloon is made on the basis of that of the time of propagation τ i.e., the time separating the signal emitted by the satellite from the signal returned by the balloon responder. In memorandum No. 6, the relation linking τ to the distance d_1 and d_2 was written in the form

$$\tau = \frac{d_1 + d_2}{c}$$

which amounted to stating that transmission takes place in a vacuum. Strictly considered, even when admitting that

the velocity of propagation is the same as the velocity of the group, the relation linking τ to $\underline{d}$ has the form

$$\tau = \frac{n_1 d_1 + n_2 d_2}{c} \neq \frac{n (d_1 + d_2)}{c}$$

since the transmitted and the received signal both have the same frequency of 400 Mc/s.

Since the group velocity Vg is equal to nc ($\underline{n}$ = local index), the time $d\tau$ required by the wave for travelling through an element ds is

$$d\tau = \frac{ds}{Vg} = \frac{1}{c} \frac{ds}{n}$$

and consequently the time τ for transmission and reception of the signal between the satellite and the balloon is

$$\tau = \frac{2}{c} \int_{S_B}^{S_S} \frac{ds}{n}$$

on the supposition, as assumed in Memorandum No. 6, that it is possible in regard to localization to replace the ellipsoid with the foci S_1 S_1' by a sphere centered at the midpoint of S_1 S_1'.

If we express the index n (index at the level of the curvilinear abscissa s) as a function of electronic density

$$n = \sqrt{1 - \frac{f_c^2}{f^2}} \simeq + 0{,}5 \frac{f_c^2}{f^2}$$

where $n = 1 - a N$ with $a = \frac{10^{-12}}{0{,}025 f^2}$

where f_c is the critical frequency (in Mc/s) and N the electronic density per m^3, we have

$$\tau = \frac{2}{c} \int_{s_B}^{s_S} ds \left(1 + \frac{1}{2} \frac{f_c^2}{f^2}\right) \quad (1)$$

$$\tau = \frac{2}{c} (s_S - s_B) + \frac{2a}{c} \int_{s_B}^{s_S} N ds$$

The first term of the second member of the expression (cf. memorandum by J.C. Thoin) represents the time of travel in vacuum as employed in memorandum No. 6. The second term corresponds to the ionospheric error whose influence is to be examined and which is equal to

$$\Delta\tau = \frac{2a}{c} \int_{s_B}^{s_S} N ds \quad (2)$$

a) It is necessary to note initially that, at the frequencies utilized (400 Mc/s), the magnetic field and the curvature of the radii have a negligible effect in regard to that produced

by the variation of the total content of electrons along the trajectory satellite-balloon.

b) Analytical integration of expression 2 is possible only at the price of the assumption of a "flat earth" which is valid provided the radii do not have too slight an inclination (cf. memorandum of J.C. Thoin) which would make it necessary to retain an angle of sight greater than 6°.

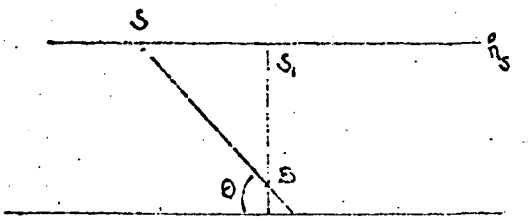
In consideration of the hypotheses a) and b) and if η is the total content of electrons at the vertical

$$\left(\eta = \int_{S_B}^{S_S} N ds \right)$$

we have

$$\Delta\tau = \frac{2a}{c} \frac{\eta}{\sin\theta}$$

where θ is the angle between the radius and the horizontal and $\underline{h}$ is the vertical ordinate.



If we base ourselves on hypothesis a) (only slightly deviated radii), we also have

$$s_S = \frac{h_S}{\sin\theta} \qquad s_B = \frac{h_B}{\sin\theta}$$

and consequently

$$\Delta\tau = \frac{2a}{c}\,\frac{\eta}{\sin\theta} \qquad \frac{\Delta\tau}{\tau} = \frac{a\eta}{h_S} \tag{3}$$

NOTE: Let us emphasize the remark made by J.C. Thoin. Since the quantity η may vary in proportions from 1 to 100, a systematic correction will not produce any appreciable improvement of accuracy.

NOTE: Since the variations of η are contained approximately between 0.5×10^{16} e/m^2 and 5×10^{17} e/m^2, we have adopted the mean value 5×10^{16} e/m^2.

As a function of these values, the errors of localization due to ionospheric effects are calculated on the basis of the data grouped in Table 1.

	minimum	mean	maximum
$\Delta\tau/\tau$	$2{,}5\ 10^{-6}$	10^{-5}	$2{,}5\ 10^{-4}$
$\Delta\tau$	$4\ 10^{-8}$	$1{,}6\ 10^{-7}$	$4\ 10^{-6}$
$\theta = 0{,}1$ rad Δd (m)	6	24	600
$\Delta\tau$	$4\ 10^{-9}$	$1{,}6\ 10^{-8}$	$4{,}10^{-7}$
$\theta = \pi/2$ Δd	0,6	2,40	60

1.2 Formulation

We saw in memorandum No. 6 ("The influence of errors in measurement of distance on the probability of localization") that the position of the base in relation to the balloon has a considerable influence on the accuracy of localization. It is evident a priori that the ionospheric effect can only accentuate this phenomenon in view of the considerable importance which it assumes at the limit of visibility. Since the preceding calculations were carried out on the hypothesis of an absolute error of distance, we have thought it preferable to retain this type of information in the present case by evaluating the errors due to a Δd linked to the ionosphere. For each of the bases utilized, has been evaluated as a function of the angle Δd in consideration of the fact that the visibility is evaluated as restricted to a minimum angle of sight of 6°.

1.2.1 Absolute errors of distance due to ionospheric effect

From the expressions

$$\Delta\tau = \frac{2a}{c}\,\frac{\eta}{\sin\theta}$$

and

$$\Delta d = \frac{c\,\Delta\tau}{2}$$

we deduce

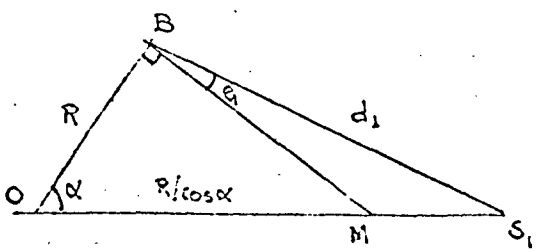
$$\boxed{\Delta d = \frac{a\eta}{\sin\theta}}$$

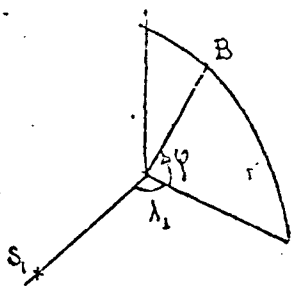
For a 400-Mc/s carrier, we have $a = 2.5 \times 10^{-16}$ and consequently $\Delta d = 2.5 \times 10^{-16} \eta / \sin\theta$. The calculations were performed for two values of

$$\eta = 5\ 10^{16} \quad \text{(mean value)} \quad \Delta d_m = 12{,}5 / \sin\theta$$

$$\eta = 5\ 10^{17} \quad \text{(maximum value)} \quad \Delta d_m = 125 / \sin\theta$$

1.2.2 Calculation of elevation





In the plane S_1 OB, we have the relations:

$$d_1^2 = R^2 + R_s^2 - 2 R R_s \cos\alpha$$

with $\cos\alpha = \cos\varphi \cos\lambda_1$

$$B M^2 = R^2 \left(\frac{1}{\cos^2\alpha} - 1 \right)$$

$$S_1M^2 = d_1^2 + B M^2 - 2 d_1 B M \cos\theta_1$$

where θ_1 perpendicular is the elevation and consequently

$$S_1M^2 = \left[R_s - \frac{R}{\cos\alpha} \right] = d_1^2 + R^2 \left(\frac{1}{\cos^2\alpha} - 1 \right) - 2 d_1 R \left(\frac{1}{\cos^2\alpha} - 1 \right)^{1/2} \cos\theta_1$$

or, by taking into account the expression of d_1:

$$R_s \left[\frac{1}{\cos\alpha} - \cos\alpha \right] = d_1 \, tg\,\alpha \, \cos\theta_1$$

$$\boxed{\cos\theta_1 = \frac{R_s \sin\alpha}{d_1}}$$

1.2.3. Ionospheric errors

We utilize here the expressions calculated in Memorandum No. 6 for giving the coordinates of the error in the plane tangential to the sphere of the balloons as a function of the position of the balloon and of the absolute errors of d_1 and d_2

$$x_E = \frac{e_1 d_1 \cos\lambda_2 - e_2 d_2 \cos\lambda_1}{R_s \sin 2\varphi}$$

$$y_E = - \frac{e_1 d_1 \sin\lambda_2 - e_2 d_2 \sin\lambda_1}{R_s \sin\varphi \sin 2\varphi}$$

with, exclusively for the ionospheric errors:

$$e_1 = \varepsilon_1 = \Delta d_1 = \frac{2{,}5\ 10^{-16}\,\eta}{\sin\theta_1}$$

and

$$e_2 = \varepsilon_2 = \Delta d_2 = \frac{2{,}5\ 10^{-16}\,\eta}{\sin\theta_2}$$

If we desire to take into account the electronic errors, we must take e_1 and e_2 in the form $e_1 + \varepsilon_1$, where ε_2 ε_1 and e_2 are the Gauss random variables already utilized and ε_1 ε_2 are the ionospheric errors. We thus obtain the general program furnished in the Annex which gives the error obtained for the different positions of the balloon.

1.3 Results

1.3.1 Influence of base position

The results for a distance from the trace of 5° are plotted in diagram 1. We find a "flat" for centered bases whereas the error increases very rapidly at the limit of visibility as will be seen from the following Table

Base Position	Error in m.	
symmetrical base	27	T = 50 s at 5° from the trace
median base	39	
limit of visibility	960	

1.3.2 Influence of time between interrogatior

Diagram 1 also shows that, for short times T, this influence is negligible at least for the median bases and at the limit of visibility.

In general, however, the ionospheric error increases with T especially for centered bases where we find the following results (case of symmetrical bases)

T	50	100	200	400
E	27 m	29 m	39 m	85 m

We see that the error varies very rapidly for large T.

1.3.3 Influence of distance from the trace

Let us turn to diagram 2 which was plotted for T = 50 sec at different distances from the trace. We see that there is little variation for short distances from the trace. However, we do note that the flat round the centered bases disappers when φ increases, for large " φ " the curves appear to become staggered (cf. 10° and 20°). The error increases appreciably with φ. We thus have the following values for symmetrical bases:

φ	5°	10°	20°
E	27 m	28 m	72 m

1.3.4 Influence of total electron content

We see in the formulas of para.§1.2.3 that the error is proportional to η. The preceding diagrams were plotted for the mean value $\eta = 5 \times 10^{16}$. We consequently find that the errors may reach a value 10 times greater and corresponding to η with a maximum of 50×10^{16}.

1.4 Influence respectively of the Ionosphere and of the Electronic Errors

Since the intrinsic ionospheric errors remain relatively low, we took the maximum value of η with the intension of obtaining characteristic results. The preceding paras make it possible to assume that the contribution of the ionosphere to the errors of localization becomes appreciable only at the limit of visibility and for intervals between interrogation and distances from the trace relatively large. Diagram 3 illustrates this result in the case of T = 50 at 5° from the trace. The presence of the ionosphere will therefore augment the character of the electronic error only in regard to the influence of the position of the base and of the distance from the trace. This contribution will moreover be as much less as the intervals between interrogation are selected shorter because of the wind.

1.5 Conclusion

The electric particles of the ionosphere therefore do not seem to modify the results of the preceeding memoranda in any appreciable manner. In fact, the resulting disturbance

remains relatively minor and becomes appreciable only in cases where the other errors are already large (slight distance from the trace, limit of visibility) and which are cases which should perhaps be eliminated.

II. ERRORS DUE TO VARIATIONS OF ALTITUDE OF THE BALLOONS

In the preceding memoranda, the balloons were assumed to remain at a known constant altitude. We here consider the distrubance in localization due to variation of balloon altitude.

2.1 Brief Review

At the present time we have only little information on these possible variations. Since the balloons are assumed to have constant density, they will displace themselves on "iso-dense" surfaces whose character is not well known. CNES suggested an investigation in order to clarify these phenomena.

In the present state, it would seem that the errors likely to occur in the evaluation of balloon altitude are of a "tansitory" nature. Actually, under permanent regime, a balloon should be found at the altitude of reference, even if it is located in a gradient of vertical wind.

We are assuming in this memorandum that the variations of altitude cannot be anticipated (and that consequently the resulting errors cannot be compensated) and that they

will not exceed a few hundred meter. This order of magnitude was suggested by CNES as a reasonable estimate.

2.2 Formulation

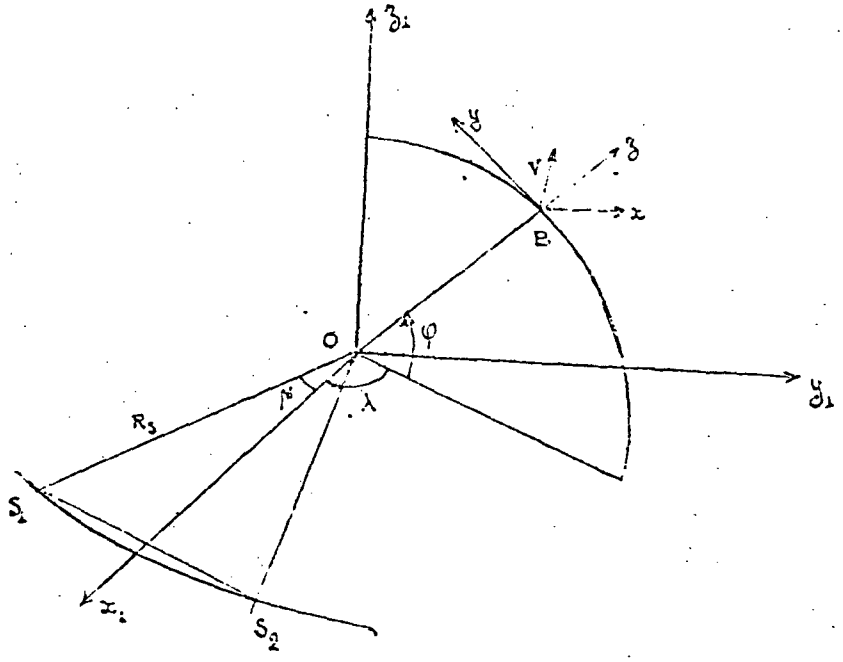
Since the errors on the radius of the sphere of the balloons are on the order of a few hundred meters, we are basing the subsequent calculations on the following approximations:

P_1 = sphere (S_1, d_1) merging with its plane tangential at B;

P_2 = sphere (S_2, d_2) merging with its plane tangential at B;

Sphere (O, R) merging with its plane tangential at B.

The problem can be resumed as follows: since a known position of the balloon B is given by the angles λ and φ on a sphere of the balloons with the assumed radius R, what would be the resulting error if the balloon were actually at a different altitude R + h.



The balloon is localized at the intersection of the planes P_1 and P_2 with the plane of the assumed balloon altitude (and therefore parallel to Bxy at the distance h) instead of lying on Bxy. Let us attempt to find the point E where the ballon is localized incorrectly:

In the axes $ox_1\ y_1\ z_1$ indicated, a vector coliner with the intersection of P_1 and P_2 is

$$\vec{V} = \overrightarrow{S_1\ B} \wedge \overrightarrow{S_2\ B}$$

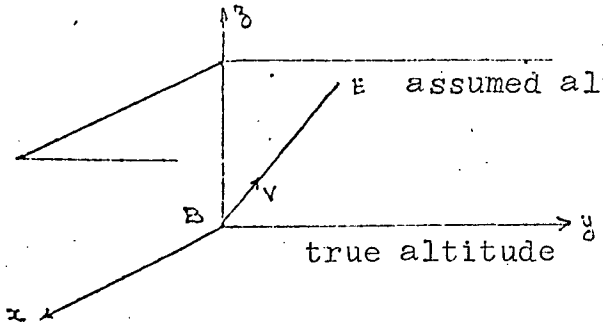
and consequently, if we express the coordinates,

$$S_1 \begin{vmatrix} R_s \cos\lambda \\ -R_s \sin\lambda \\ 0 \end{vmatrix} \qquad S_2 \begin{vmatrix} R_s \cos\lambda \\ R_s \sin\lambda \\ 0 \end{vmatrix} \qquad B \begin{vmatrix} R\cos\varphi\ \cos\lambda \\ R\cos\varphi\ \sin\lambda \\ R\sin\varphi \end{vmatrix},$$

and

$$\vec{V} \begin{vmatrix} 2\,R\,R_s \sin\lambda \sin\varphi \\ 0 \\ 2\,R_s^2 \sin\lambda\cos\lambda - 2R\,R_s \cos\varphi \sin\lambda \cos\lambda \end{vmatrix} \quad \text{we take } (x_1 y_1 z_1) \quad \begin{vmatrix} R\sin\varphi \\ 0 \\ R_s\cos\lambda - R\cos\varphi\cos\lambda \end{vmatrix}$$

E is the point of intersection of the straight line $(B,\vec{V})$ with the assumed plane of altitude.



The error BE is therefore obtained by writing that the point E is at the altitude h. For the trihedron xyz linked to the balloon, we have

$$\text{in } \begin{matrix} B_z \\ x_1 y_1 z_1 \end{matrix} \left| \begin{matrix} \cos\varphi & \cos\lambda \\ \cos\varphi & \sin\lambda \\ \sin\varphi & \end{matrix} \right.$$

Let ρ_E be the coefficient of proportionallity giving $\vec{BE}$ on the basis $\vec{V}$, we then have the condition

$$z_E = \rho_E (\vec{V}. \vec{B}_z) = \rho_E R_S \cos\varphi \sin\varphi$$

We therefore have

$$\rho_{\varepsilon} = \frac{h}{R_s \cos\gamma \sin\varphi}$$

$$E = \rho_{\varepsilon} |(V)| \qquad \text{or} \qquad E = \frac{h\sqrt{(R\sin\varphi)^2 + (R_s\cos\gamma - R\cos\varphi\cos\lambda)^2}}{R_s \cos\gamma \sin\varphi}$$

It should be noted that this error is always greater than the error of altitude h.

Let us attempt to find the extrema:

$$\frac{dE}{E} = \frac{(R_s \cos\gamma - R\cos\varphi\cos\lambda)\, R\cos\varphi\sin\lambda}{|V|}$$

From which $\alpha = \gamma$ with symmetrical bases,

$$\cos\lambda = \frac{R_s\cos\gamma}{R\cos\varphi} \quad \text{with the condition } \cos\varphi > \frac{R_s}{R}\cos\gamma$$

(always outside of visibility)

and consequently the computer program furnished in the Annex.

In order to associate all of the electronic, the ionospheric and the balloon-altitude errors, let us introduce

the coordinates of $\vec{BE}$. We then have by taking into account

$$B_x \left| \begin{array}{l} -\sin\lambda \\ +\cos\lambda \\ 0 \end{array} \right. \qquad B_y \left| \begin{array}{l} -\sin\varphi\cos\lambda \\ -\sin\varphi\sin\lambda \\ \cos\varphi \end{array} \right.$$

$$x_E = \rho_E (\vec{V}.\vec{B}_x) = -\frac{h\,R\sin\lambda}{R_s\cos\varphi}$$

$$y_E = \rho_E (\vec{V}.\vec{B}_y) = \frac{h\left[R_s\cos\varphi\cos\varphi - R\cos\lambda\right]}{R_s\sin\varphi\cos\varphi}$$

$$z_E = \rho_E (\vec{V}.\vec{B}_z) = h$$

2.3 Results

2.3.1 Influence of base position

The curves of diagram 4 illustrate the results obtained. It will be noted that this error increases when

we approach the limit of visibility. This influence is as much less as we are further away from the trace.

2.3.2 Influence of distance from the trace

Diagram 4 also shows that the error diminishes rapidly when we move away from the trace. It also becomes almost independent of the position of the base. For the symmetrical bases, we have in fact the following results

φ	5°	10°	20°
E in m	170	120	102

h = 100 m

2.3.3 Influence of interval between interrogation

This influence has little importance to the extent in which we are far from the trace so that we arrive at the following table (symmetrical bases):

T	50	100	200	300	400
5°	169,4	168,6	165,5	160,4	153,2
20°	101,6	101,6	101,4	101,2	100,9

h = 100 m

It will also be noted that the error diminishes slightly when the time T increases.

2.3.4 Influence of error of altitute h

The approximation made in establishing the formula of para. 2.2 led us to adopt a simple proportionality. Consequently the above results are given only for the case h = 100 m. Let us note that this approximation rests valid except at very short distances from the trace where the magnitude of the errors found renders the consideration of the tangential planes invalid or pointless ("caduque").

2.4 Conclusion

We see that the errors due to variations of balloon altitude further reduce the accuracy at the limit at visibility and for short distances from the trace. This contribution appears to remain limited relative to the total errors, provided the amplitude of the variations of altitude is on the order of magnitude selected. Let us note that the minor influence of the interval between interrogation makes possible optimum selection of the latter in order to minimize the other types of error.

III. ERRORS DUE TO INACCURACIES IN THE EPHEMERIDES

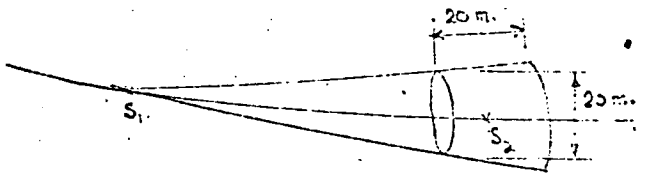
3.1 Brief Review

The study in this section was undertaken on the basis of the CNES memorandum on errors in the ephemerides. Localization is made from two positions of the satellite are (t_1, d_1) and (t_2, d_2), the passage of t_1 to the position

S_1 on the basis of the ephemerides will therefore affect the localization of the balloon if there is an inaccuracy in the ephemerides. We can in fact distinguish -

(1) an error in the position of S_1. From the results of the large number of interferometer measurements, we can assume that the true position of S_1 is found within the "parallelepiped" with dimensions of 280 m parallel to the trace, 120 m perpendicular to the trace and 80 m radial; and on the other hand,

(2) an error in the velocity vector $\vec{V}$ of modulus 7.6×10^{-5} km/sec or ΔV, and of direction 2×10^{-5} radian or $\Delta\varphi$. With the position of S_1 established, S_2 is therefore found in a small volume of calculable dimensions.



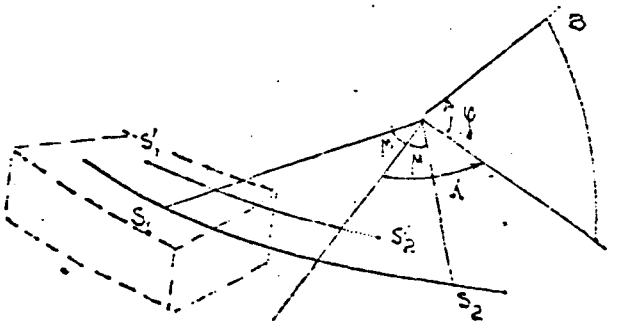
However, we assume in the following the error "$\Delta\varphi$" as negligible which constitutes an increase in the inaccuracy of S_2 of 10 m in the three dimensions.

On the other hand, we are utilizing here the case examined in the preceding memoranda, i.e., an orbit of

900 km by arbitrarily applying to this the above data relative to a satellite altitude of 500 km. It will therefore be necessary to re-adjust these values in an accurate quantitative examination of the phenomenon.

3.2 Formulation

In order to account for these errors, it would therefore be necessary in theory to return to a localization in an inertial trihedron and not in relation to the orbit. However, we can replace the inaccuracies in the positions of S_1 and S_2 as error in the distances d_1 and d_2. Localization is then made in relation to the plane of the orbit from the assumed positions of S_1 and S_2 and from distances corrected by the quantities e_1 and e_2 calculated as below



On the hypothesis that we neglect the error in the direction of the velocity vector (trajectory S_1' S_2'

parallel to the theoretical orbit), we can introduce ε_λ, ε_μ, ε_ν, ε_φ in such manner that:

ε_λ = error in λ = inaccuracy in S_1 parallel to the orbit;

ε_u = error in "$R_{S/R}$" = inaccuracy in the radius of the orbit;

ε_φ = error in φ = inaccuracy in S_1 perpendicular to the orbit;

ε_ν = error in ν = 1/2 error in S_1 S_2. We can then write:

$$d_1^2 = 1 + u^2 - 2u \cos\varphi \cos(\lambda + \nu)$$

$$e_1 d_1 = \left[u - \cos\varphi \cos(\lambda + \nu)\right]\varepsilon_u + u \sin\varphi \cos(\lambda + \nu)\varepsilon_\varphi + u \cos\varphi \sin(\lambda+\nu)\varepsilon_\lambda + u\cos\varphi \cos(\lambda+\nu)\varepsilon_\nu$$

and the same for e_2 by changing ν into $-\nu$

We will utilize the formulas established in para. 4.1.1 of memorandum No. 6 giving the errors of localization in the tangential plane as a function of e_1, e_2 and of the position of the balloon (λ, φ) :

$$x_E = \frac{e_1 d_1 \cos(\lambda-\nu) - e_2 d_2 \cos(\lambda+\nu)}{u \sin 2\nu} \quad \text{and} \quad y_E = \frac{e_1 d_1 \sin(\lambda-\nu) - e_2 d_2 \sin(\lambda+\nu)}{u \sin\varphi \sin 2\nu}$$

We therefore have here: $x_\varepsilon = \varepsilon_u \frac{\sin\lambda}{\cos\nu} + \varepsilon_\lambda \cos\varphi + \varepsilon_\nu \frac{\cos\varphi \sin 2\lambda}{\sin 2\nu}$

$$y_\varepsilon = -\varepsilon_u \left[\frac{u\cos\lambda - \cos\varphi\cos\nu}{u\cos\nu \sin\varphi}\right] - \varepsilon_\varphi + \varepsilon_\nu \left[\frac{\cos\varphi(\cos 2\nu - \cos 2\lambda)}{\sin 2\nu \sin\varphi}\right]$$

Reproduced from best available copy.

Reproduced from best available copy.

Let the modulus be:

$$\bar{E}^2 = \bar{x}^2_E + \bar{y}^2_E \text{ , by steting } \bar{\varepsilon}^2_U = \sigma^2_U$$

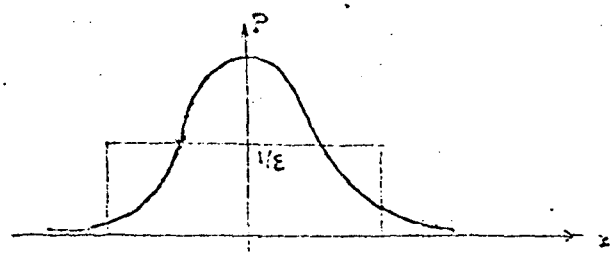
since the errors ε_U, ε_λ, ε_φ, ε_μ are independent

$$\bar{\varepsilon}^2 = \frac{\sigma_u^2}{\cos^2\mu}\left[\sin^2\lambda + \frac{(U\cos\lambda - \cos\varphi\cos\mu)^2}{U^2\sin^2\varphi}\right] + \sigma_\lambda^2\cos^2\varphi + \sigma_\varphi^2 + \frac{\sigma_\mu^2\cos^2\varphi}{\sin^2 2\mu}\left[\sin^2 2\lambda + \frac{(\cos 2\mu - \cos 2\lambda)^2}{\sin^2\varphi}\right]$$

It will therefore not be surprising to find in the preceding expression the quantity $\sigma_\varphi^2 + \sigma_\lambda^2\cos^2\varphi$ which we know to represent the error due to ε_φ and ε_λ in the plane tangential to the balloon.

Numerical Treatment

A computation program is furnished in the Annex in which are considered the values ε_λ, ε_μ .... etc., as resulting from normal laws taking into account as well as possible an equal probability of finding S in the parallelepiped defined in para 3.1.



By identifying the moments of order 2, we have

$$\sigma^2 = \int_{-\infty}^{\infty} \frac{1}{\varepsilon} x^2 dx = \frac{\varepsilon^2}{3}$$

and therefore take $\sigma_\lambda = \frac{\sqrt{2}}{\sqrt{3}}\varepsilon_\lambda$; ...

3.3 Results

3.3.1 Influence of base position

Diagram 5 illustrates the results for an interval between interrogation of 50 sec and of different distances from the trace. We see that the error is a minimum for summetrical bases ($\lambda = o$) and increases at the limit of visibility. This increase is moreover less when we move away from the trace.

	symmetrical bases	limit of visibility
$\varphi = 5°$	0,17 km	8 km
$\varphi = 20°$	0,12 km	1,5 km

3.3.2 Influence of distance from the trace

Diagram 5 also shows that, with symmetrical bases, the distance from the trace does not come into play but that, generally, the error decreases appreciably for median bases and at the limit of visibility when the balloon is located further away from the trace.

3.3.3 Influence of the interval between interrogation

Diagram 6 shows the influence of this important parameter. We find the appearance of the minimum as a function of the position of the base for large T and this

minimum is displaced toward the limit of visibility when T increases. We therefore see that, in the case of relatively low T as utilized in practice, this influence is primarily of significance at the limit of visibility where the error diminishes when T increases.

3.4 Conclusions

Let us remember the preceding results must be considered as orders of magnitude and show that we are in the presence of a cause of large error comparable to the electronic errors of measurement. This will not surprising when we consider that the most erroneous estimate of the position of S_1 corresponds to an error of distance of more than 300 m. It will therefore be advantageous to establish ephemerides prepared from very accurate measurements of the orbital parameters over a large number of passages.

IONOSPHERIC ERRORS

PAF

$_i$=2 D G E

1 Read R S H T N S

2 Print with 3 DEC TAB R TAB Z TAB H TAB T TAB N TAB S RC

3 Calculate V = $\sqrt{(398599/R^3)}$

4 Calculate 0 = Z (COS H)^2/R+(SIN H)$\sqrt{(1-Z^2(\text{COS H})^2/R^2)}$

5 State F = 0,087267

6 Calculate W = ZF

7 Print with 3 DEC TAB W TAB F RC

8 Calculate C = ARC COS (0/(COS F))

9 State L = 0

10 Calculate A = COS (L+VT/2)

11 Calculate B = COS (L-VT/2)

12 Calculate D_1 = $\sqrt{(Z^2 + R^2 - 2ZRA\ (\text{COS F}))}$

13 Calculate D_2 = $\sqrt{(Z^2 + R^2 - 2ZRB\ (\text{COS F}))}$

14 Calculate G_1 = R ($\sqrt{(1-A^2\ (\text{COS F})^2)}$)/$D_1$

15 Calculate G_2 = R ($\sqrt{(1-B^2\ (\text{COS F})^2)}$)/$D_2$

16 Calculate E_1 = 0,0025N/ ($\sqrt{(1-G_1^2)}$)

17 Calculate E_2 = 0,0025N/$\sqrt{(1-G_2^2)}$

18 Calculate X = S (D_1^2 B^2 + D_2^2 A^2) + (E_1E_1 B - E_2 D_2 A)^2

19 Calculate Y = (S(D_1^2 (1-B^2) + D_2^2 (1 - A^2))
+(E_1D_1(SIN (L-VT/2)-R_2D_2(SIN(L+VT/2)))^2/
(SIN F)^2

20 Calculate E_o = ($\sqrt{(X + Y)}$)/R (SIN VT)

21 Print with 5 DEC TAB TAB TAB L TAB E_o TAB E_1 TAB E_2 RC RC

22 Make L = L + 50V

23 If > C go to 25

24 Go to 10

25 Make F =2F

26 If F> 0,4 go to 1

27 Go to 6

END

R: " R_s " Radius or orbit

H: " ϵ " Elevation

F,W: φ, γ Distance from trace

C: " γ " 1/2 Arc of visibility

L: " λ " Position of base

D_1, D_2: " d_1 "," d_2 "

E_1, E_2: Errors in distances

E_o: Error of localization

Z: Radius of balloon sphere

T: Interval between interrogation

Radius, KM

G_1, G_2: " $\cos\hat{e}_1$ "," $\cos\hat{e}_2$ "

ERROR DUE TO VARIATIONS OF BALLOON ALTITUDE

```
PAF
V = √(398599/R^3)
U = R/Z
1  Read R Z H T
2  Calculate U V
3  State F = 0,087267
4. Calculate C = ARC COS (1/U COS F)
5  Print with 3 DEC RC TAB F TAB T RC
6  State L = 0
7  Calculate E = H (√((SIN F)^2 + (U COS VT/2-(COS F)(COS L))^2))
                                        /U(SIN F)(COS VT/2)
8  Print with 5 DEC TAB TAB E
9  Calculate A = U (COS VT/2)/(COS F)
10 If A >1 go to 13
11 Calculate N = ARC COS A
12 Print with 5 DEC TAB M RC
13 Make L = L + 50V
14 If 1> C go to 16
15 Go to 7
16 Make F = 2F
17 If F> 0,4 go to 1
18 Go to 4
END
```

EXPLANATION OF LETTER SYMBOLS

R:" R_s "Radius of orbit

Z:" R "Radius of balloon sphere

V:" Ω_s "Rotational velocity of satellite

C:" γ "1/2 arc of visibility

L:" λ "Position of base

H:" ρ_R "Error in altitude of balloon

E:" ε "Error of localization

M: Position of an extremum

ERRORS DUE TO INACCURACIES IN EPHEMERIDES

PAF

$_{i} = 2\ E$

$_{i} = 4\ S$

$V = \sqrt{(398599/R^3)}$

$U = R/z$

$S_1 = 2\ (0,08)^2/3R^2$

$S_2 = 2\ (0,012)^2/3R^2$

$S_3 = 2\ (0.28)^2/3R^2$

$S_4 = 2\ (0,12)^2/3R^2$

1 Read R Z J T

2 Print with 3 DEC TAB R TAB T RC

3 Caltulate V U S_1 S_2 S_3 S_4

4 Calculate $0 = Z\ (COS\ J)^2/R+(SIN\ J)\ \ (1-Z^2(COS\ J)^2/R^2$

5 State F = 0,087267

6 State L = 0

7 Print with 3 DEC TAB F RC

8 Calculate A = COS F

9 Calculate C = ARC COS (0/A)

10 Calculate $E_1 = ((SIN\ L)^2 + (U\ COS\ L-A\ COS\ VT/2)^2/U^2(1-A^2))/(COS\ VT/2)^2$

11 Calculate $E_2 = A^2\ ((SIN\ 2L)^2 + (COS\ VT - COS\ 2L)^2/(1-A^2))/(SIN\ VT)^2$

12 Calculate $E_o = Z\sqrt{(S_1\ E_1 + S_2\ E_2 + S_3\ A^2 + S_4)}$

13 Print with 4 DEC TAB TAB TAB L TAB E_o RC

14 Make L = L + 50 V

15 If I > C go to 17

16 Go to 10

17 Make F = 2F

18 If F > 0,4 go to 1

19 Go to 6

END

EXPLANATION OF LETTER SYMBOLS

R:" R "Radius of orbit

Z:" R "Radius of balloon sphere

J:" ϵ "Elevation

S_1, S_2, S_3, S_4: "σ_u" "σ_μ" "σ_λ" "σ_φ"

F:" φ "Distance from trace

L " λ "Base position

C:" δ "1/2 arc of visibility

E_o: Error of localization

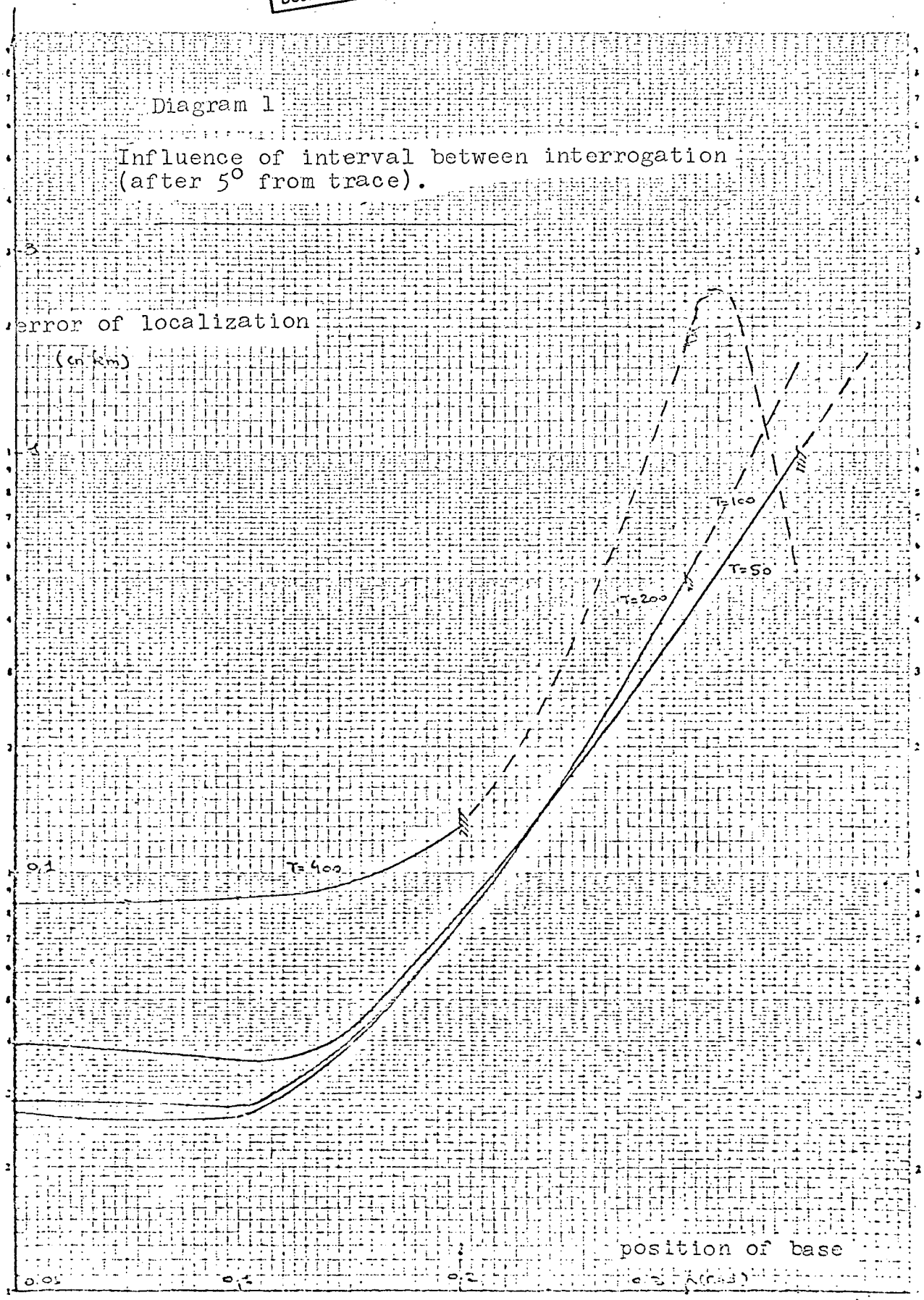
Reproduced from best available copy.

Tochon-Lepage, Paris - Editeur.

Quadrille Semi-Logarithmique à 3 modules

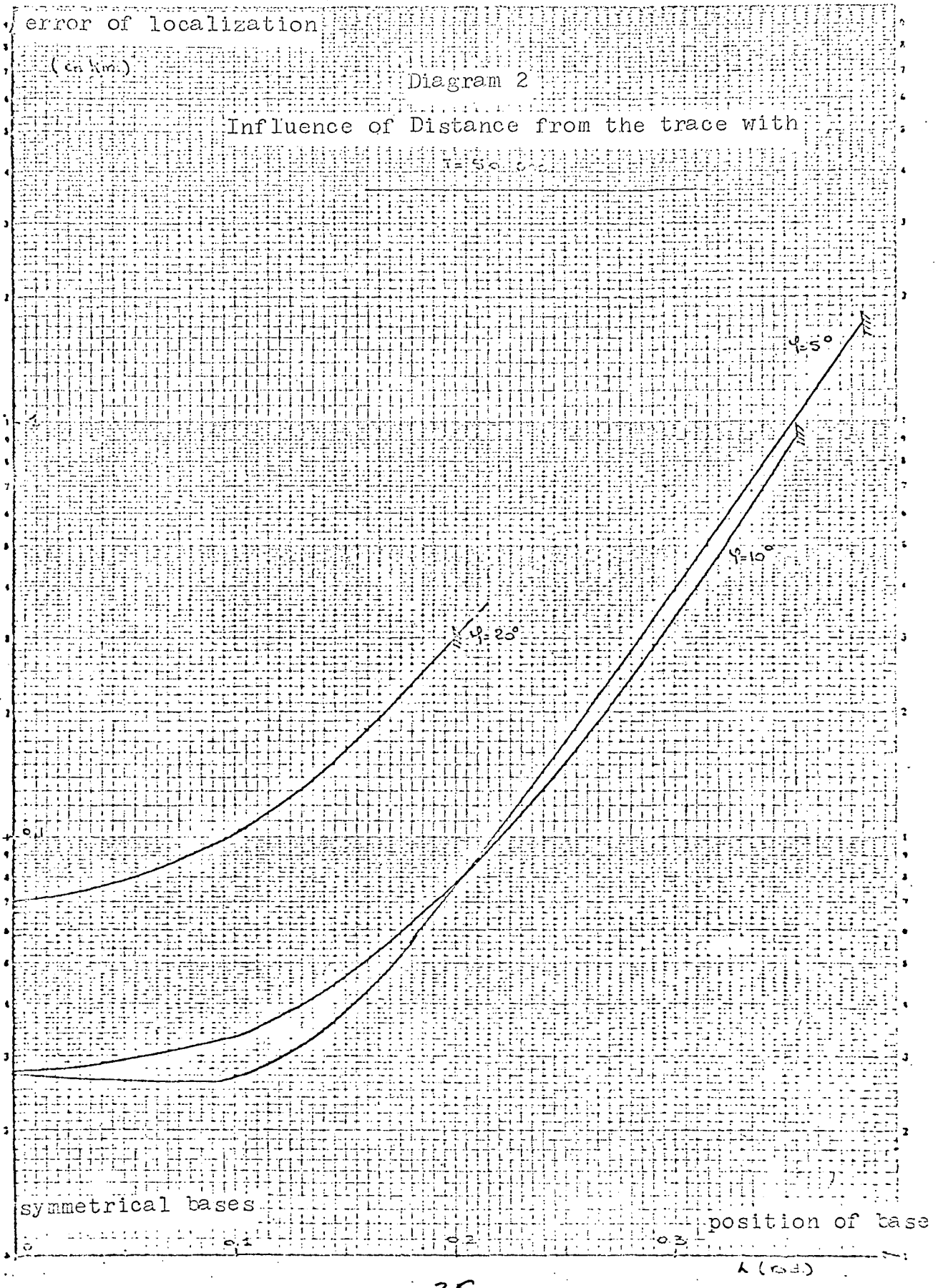
Diagram 1

Influence of interval between interrogation (after 5° from trace).

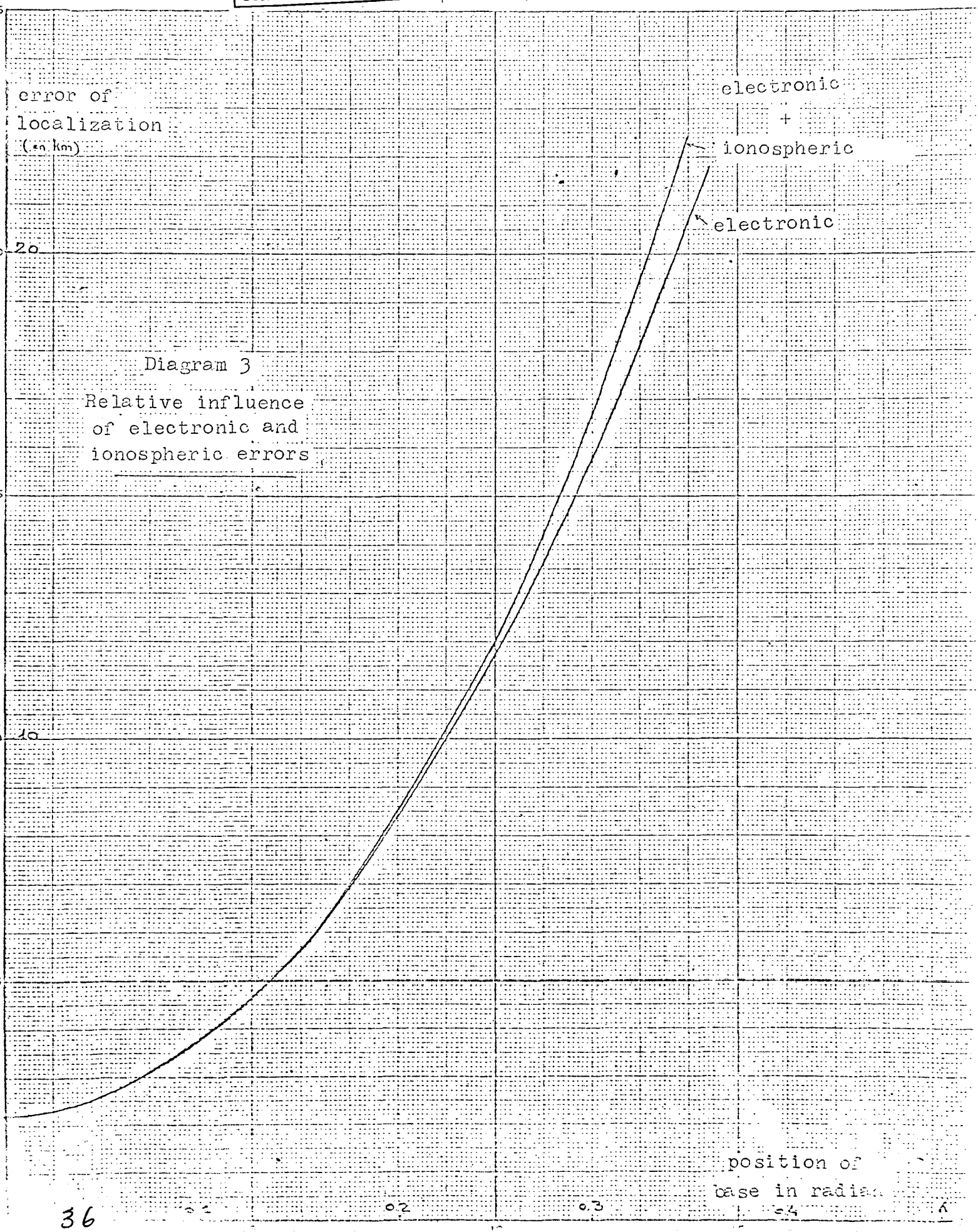


Tochon-Lepage, Paris - Editeur.

Quadrillé Semi-Logarithmique à 3 modules



Reproduced from
best available copy.



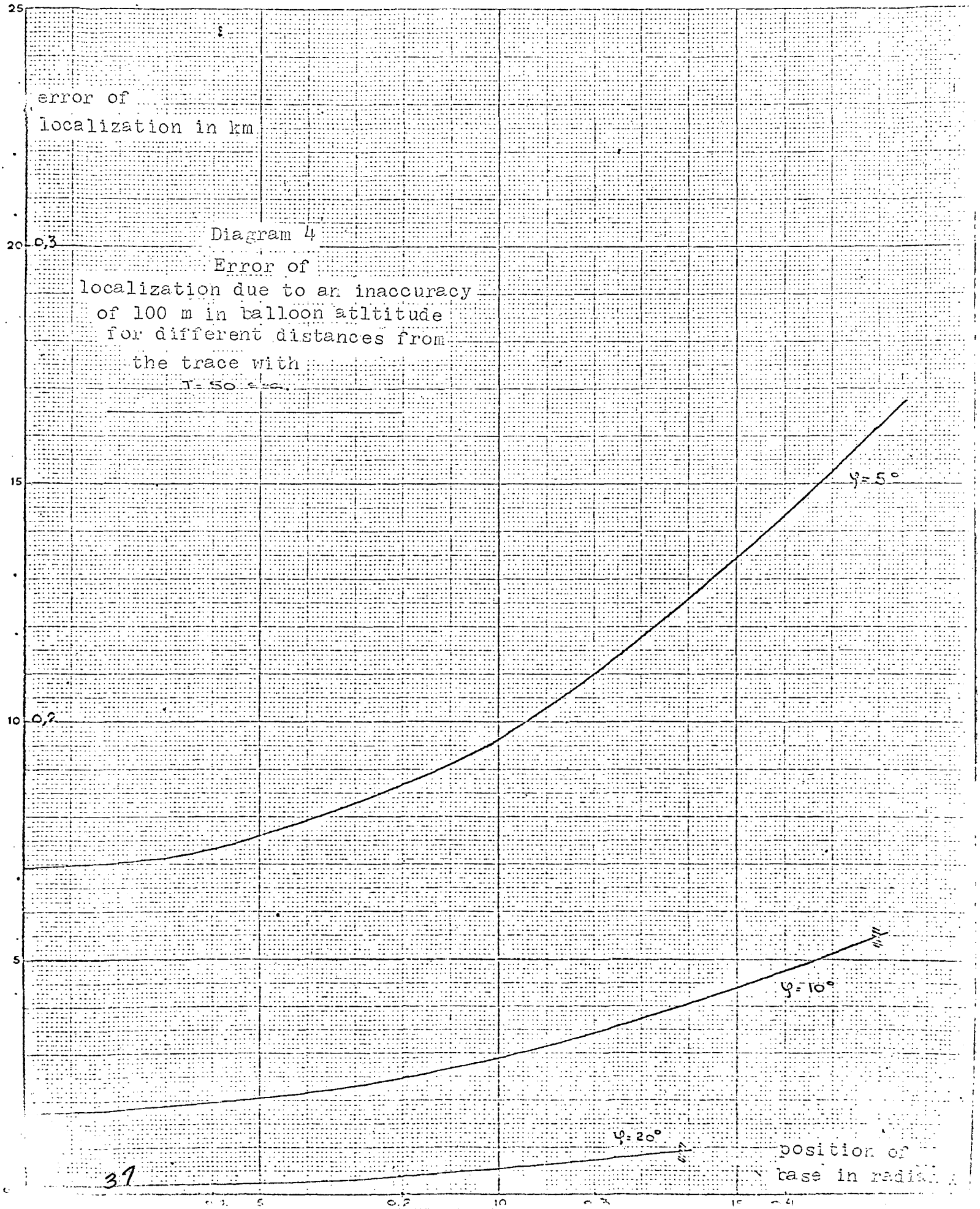


Diagram 4

Error of localization due to an inaccuracy of 100 m in balloon atltitude for different distances from the trace with T = 50 sec.

Reproduced from best available copy.

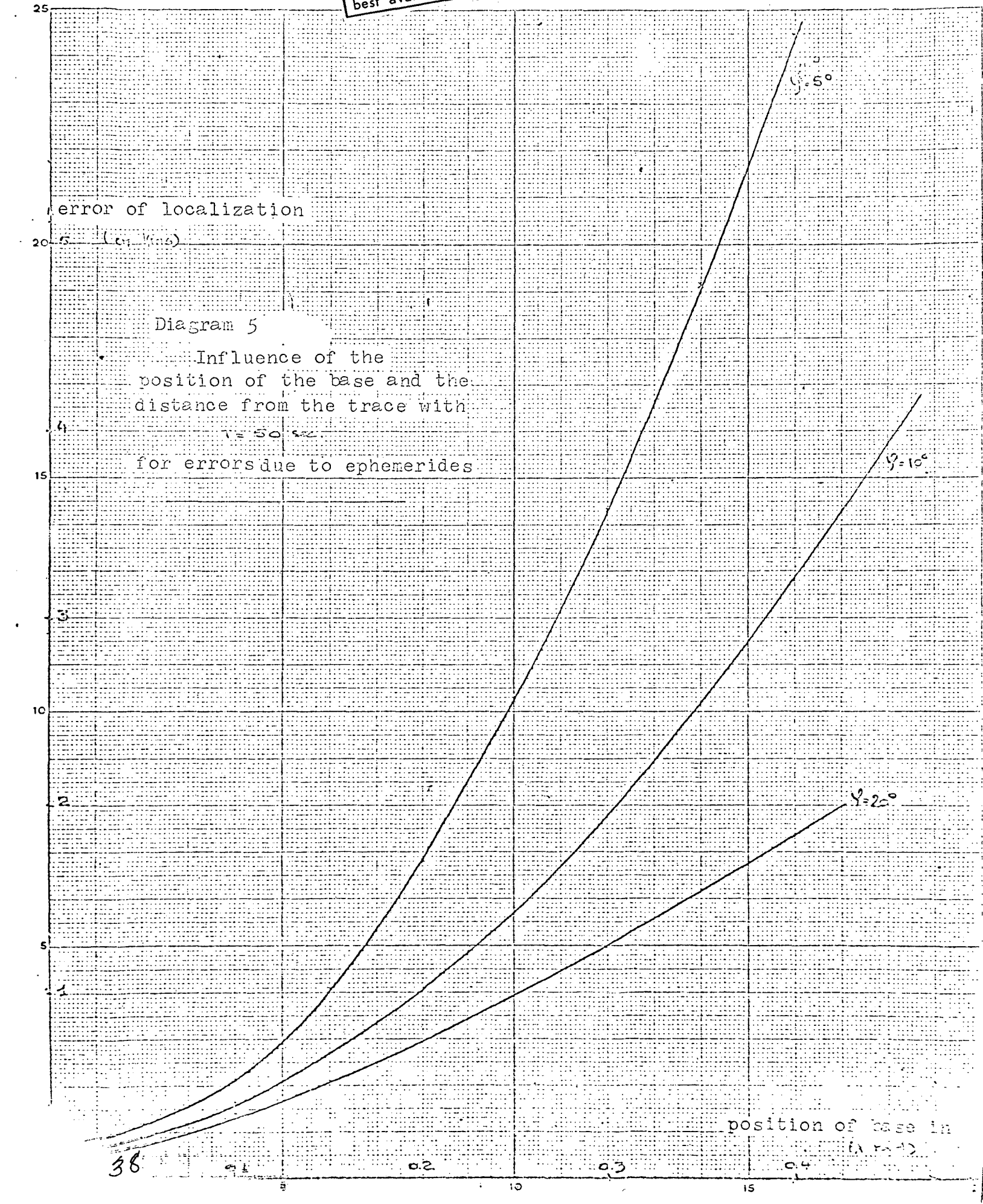


Diagram 5

Influence of the position of the base and the distance from the trace with $\tau = 50$ sec for errors due to ephemerides

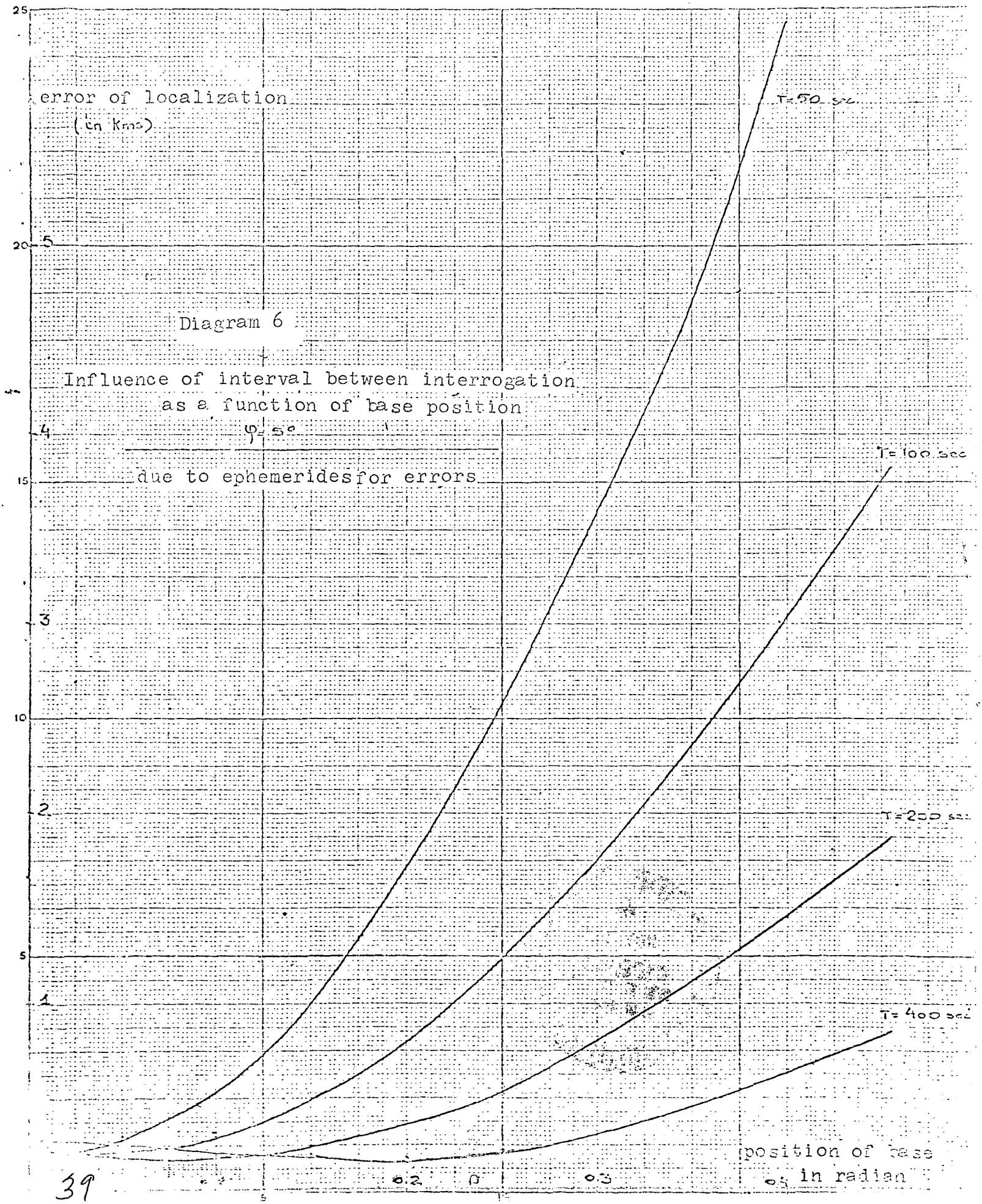


Diagram 6

Influence of interval between interrogation
as a function of base position
$\varphi = 5°$

due to ephemerides for errors